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RADIATIVE–CONDUCTIVE HEAT TRANSFER IN TEMPERATURE-WAVE CONDITIONS

S. N. Kravchun and L. P. Filippov

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Relations describing the radiative–conductive heat transfer in plane temperature-wave conditions are obtained. An analysis is given of the effect of radiative transfer on the measurement of the thermophysical properties of the material by the method of a regular thermal mode of the third kind.

In considering heat-transfer mechanisms in liquids and compressible gases, the role of the radiative transfer is an important and little-studied problem. It is known that heat transfer by radiation may make a notable contribution to the heat conduction of a liquid even at room temperature, and its role increases greatly with increase in temperature [1]. To obtain information on radiative transfer from steady-state experiments, it is necessary to make measurements in cells of different size, which is very troublesome. The potential of nonsteady methods of investigation is fundamentally greater. In [2, 3], the role of radiative heat transfer in experiments on the probing of liquids by heat pulses was investigated; it was shown that in the early stages of this process radiant energy transfer plays a small role, and such experiments allow the pure heat conduction of liquids to be determined. In the present work, the question of radiative heat transfer is investigated in the context of liquid probing by plane temperature waves, which is the main method of measuring the thermal-activity coefficient [1].

A solution is obtained for the problem of the heat transfer in a periodically heated plane layer (represented experimentally as a metal foil) situated in a semitransparent medium. The foil constitutes an infinite plane yz, is situated at the coordinate origin (x = 0), and has a specularly reflecting surface. The heat-transfer equation of the foil in the medium, taking radiation into account, is

$$\frac{W}{s}=\frac{cm}{s}\frac{\partial T_1}{\partial t}-2\lambda\frac{\partial T(0)}{\partial x}+2q(0). \tag{1}$$

The radiant energy flux to an absorbing, nonscattering medium whose optical properties are independent of temperature is given by the relation [4]

$$q(x)=2(1-R)\sigma n^2T_1^4E_3(\alpha x)+2R\sigma n^2\alpha\int_0^\infty T^4(\xi)E_2(\alpha(x+\xi))\,d\xi$$
$$+2\sigma n^2\alpha\left[\int_0^x T^4(\xi)E_2(\alpha(x-\xi))\,d\xi-\int_x^\infty T^4(\xi)E_2(\alpha(\xi-x))\,d\xi\right], \tag{2}$$

where $E_3(\alpha x)$ and $E_2(\alpha x)$ are integroexponential functions [5].

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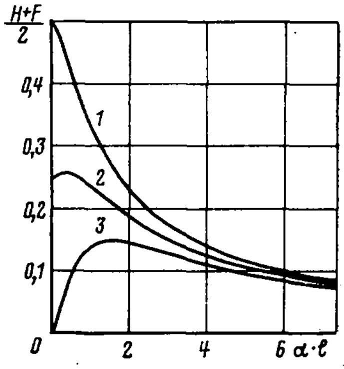

Fig. 1. Dependence of (H + F)/2 on the Knudsen number for different reflective indices of the foil—sensor: 1) R = 0; 2) 0.5; 3) 1.

To find the radiative and conductive fluxes from the foil surface, it is necessary to know the temperature distribution in the medium, which involves solving the conductive—radiative energy transfer in the medium [4]

$$\lambda \frac{\partial^2 T}{\partial x^2} - \rho c_p \frac{\partial T}{\partial t} = \frac{\partial q}{\partial x}, \quad 0 \leqslant x < \infty. \tag{3}$$

Under the action of a source with a variable component of power $W = W_0 + W_0 \exp(2i\omega t)$, the temperature of the foil and the medium contains a component that varies periodically with time; thus, for the foil, $T_1 = \bar{T}_1 + \tilde{T}_1 \exp(2i\omega t)$. Consider the case of small temperature pulsations. The smallness condition $\tilde{T} \ll \bar{T}$ is usually well satisfied in the given experiment, and $\tilde{T}$ is a fraction of a degree. This means that Eqs. (1) and (3) can be linearized with respect to $\tilde{T}(x)$.

The solution for the constant component is taken in the form $\bar{T}(x) = \bar{T}_1 = \text{const}$, which satisfies the equation for the constant component of the temperature with boundary conditions $\bar{T}(0) = \bar{T}(\infty) = \bar{T}_1$.

The method of successive approximation is used to solve Eq. (3) for the amplitude of the temperature pulsations $\tilde{T}(x)$. The zero approximation adopted is the solution of Eq. (3) without taking radiation into account (a solution of temperature-wave type)

$$\tilde{T}(x) = \tilde{T}_1 \exp\left(-\frac{x}{l}(1+i)\right), \tag{4}$$

where $l = \sqrt{\lambda / c_p \rho \omega}$ is the damping length of the temperature wave (the wave amplitude is decreased by a factor of e at $x = l$). The small-parameter coefficient appearing in the integral term of Eq. (3), which is neglected in the zero approximation, is taken to be the Biot number Bi, which characterizes the order of magnitude of the ratio between the radiant and conductive energy fluxes. In the conditions of the given experiment, $\text{Bi} \lesssim 10^{-2}$, which means that it is sufficient to consider the first approximation in the solution; the second approximation takes account in the solution of terms of higher-order smallness — of the order of $(\text{Bi})^2$.

Substituting Eq. (4) into the right-hand side of Eq. (3) for the pulsational component of the temperature gives

$$\begin{aligned} \frac{\partial^2 \tilde{T}(x)}{\partial x^2} - \frac{\rho c_p}{\lambda} 2i\omega \tilde{T}(x) = \frac{8\sigma n^2 \alpha \bar{T}_1^3}{\lambda} \Big[2\tilde{T}_1 \exp\left(-\frac{x}{l}(1+i)\right) \\ - (1-R)\tilde{T}_1 \exp(-\alpha x) - (1-R)\tilde{T}_1 \alpha x \,\text{Ei}(-\alpha x) + \tilde{T}_1 \frac{\alpha l}{1+i}(1+R) \\ \times \text{Ei}(-\alpha x) + \tilde{T}_1 \frac{\alpha l}{1+i} \ln\left(\frac{\alpha l - 1 - i}{\alpha l + 1 + i}\right) \exp\left(-\frac{x}{l}(1+i)\right) \\ - \tilde{T}_1 \frac{\alpha l}{1+i} \exp\left(-\frac{x}{l}(1+i)\right) \text{Ei}\left(-\frac{\alpha l - 1 - i}{l} x\right) - \tilde{T}_1 R \frac{\alpha l}{1+i} \\ \times \exp\left(\frac{x}{l}(1+i)\right) \text{Ei}\left(-\frac{1+i+\alpha l}{l} x\right), \end{aligned} \tag{5}$$

where $\text{Ei}(-\alpha x)$ is the integral index function of [5]. In this approximation, the radiant energy flux from the foil, given by Eq. (2), takes the form

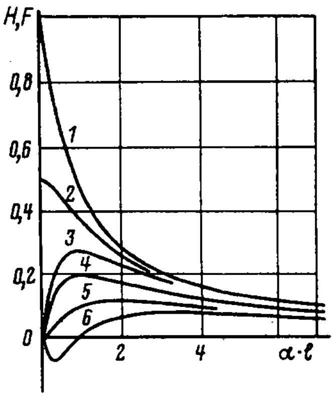

Fig. 2

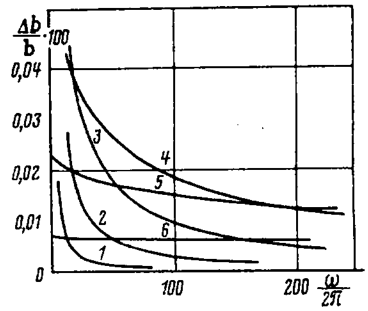

Fig. 3

Fig. 2. Dependence of F (1, 2, 3) and H (4, 5, 6) on the Knudsen number for different foil reflective indices: 1, 4) R = 0; 2, 5) 0.5; 3, 6) 1.

Fig. 3. Frequency dependence of the relative error in thermal-activity calculation due to neglecting energy transfer, $\Delta b/b$, for different absorption coefficients: 1) $\alpha = 3\cdot 10^3\ \mathrm{m}^{-1}$; 2) 10^4; 3) $3\cdot 10^4$; 4) 10^5; 5) $3\cdot 10^5$; 6) 10^6; total-reflection coefficient (R = 1). $\omega/2\pi$, Hz.

$$\tilde{q}(0) = 4\sigma n^2 \bar{T}_1^3 \tilde{T}_1 (1-R)\left\{1 - \alpha l + (\alpha l)^2 \operatorname{arctg}\frac{1}{1+\alpha l} + i\left[\alpha l - (\alpha l)^2 \ln\sqrt{2/(\alpha l)^2 + 2/\alpha l + 1}\right]\right\}. \tag{6}$$

The general solution of the inhomogenous linear Eq. (5) may be written in quadratures [6]. The solution satisfying the boundary conditions $\tilde{T}(0) = \tilde{T}_1$ and $\tilde{T}(x) \to 0$ as $x \to \infty$ yields the following expression for the conductive heat flux from the foil

$$-\lambda\frac{\partial \tilde{T}(0)}{\partial x} = -\lambda \tilde{T}_1 \frac{1+i}{l} - 8\sigma n^2 \alpha \bar{T}_1^3 \tilde{T}_1 \frac{l}{1+i} R\left[1 + \frac{\alpha l}{1+\alpha l + i} - 2\frac{\alpha l}{1+i}\ln\left(\frac{\alpha l + 1 + i}{\alpha l}\right)\right]. \tag{7}$$

Note that when R = 0 (an absolutely black wall) the presence of radiant energy transfer does not change the heat flux due to heat conduction in the first approximation.

Taking into account Eqs. (6) and (7), the amplitude of the pulsations of the total heat flux Q is

$$|Q| = \left|-\lambda\frac{\partial \tilde{T}(0)}{\partial x} + \tilde{q}(0)\right| \approx \lambda\frac{\sqrt{2}\,|\tilde{T}_1|}{l}\left[1 + \mathrm{Bi}\,\frac{H+F}{2}\right], \tag{8}$$

where

$$H = (1-R)\left[\alpha l - (\alpha l)^2 \ln\sqrt{2/(\alpha l)^2 + 2/\alpha l + 1}\right] - R\left[\alpha l + \alpha l\,\frac{2\alpha l + (\alpha l)^2}{(\alpha l)^2 + 2\alpha l + 2} - 2\alpha l \operatorname{arctg}\frac{1}{\alpha l + 1}\right], \tag{9}$$

$$F = (1-R)\left[1 - \alpha l + (\alpha l)^2 \operatorname{arctg}\frac{1}{1+\alpha l}\right] + R\alpha l\left[1 + \frac{(\alpha l)^2}{(\alpha l)^2 + 2\alpha l + 2} - 2\alpha l \operatorname{arctg}\frac{1}{1+\alpha l}\right]. \tag{10}$$

In Fig. 1, $(H + F)/2$ is shown as a function of αl for different values of R. Analysis of these curves leads to the following conclusions.

1) For a wall with a reflective index greater than ~0.5, the energy flux due to radiation may exceed the radiant flux through a transparent medium.

2) When αl ~10, the emissivity of the wall has practically no effect.

3) The radiant flux cannot exceed the radiant flux in a transparent medium from an absolutely black wall. The upper limit on the proportion of radiation in the total heat flux is Bi/2.

Note that analogous conclusions apply to steady radiant heat transfer [1].

The well-known limiting cases follow from Eqs. (8)-(10).

For a transparent medium ($\alpha = 0$), the radiant-flux amplitude is

$$|\tilde{q}(0)| = 4\sigma n^2 \overline{T}_1^3 |\tilde{T}_1| (1-R) \tag{11}$$

(this formula corresponds to the pulsations of the radiation from a foil surface in vacuum).

In the other limiting case — a medium with large absorption ($\alpha \to \infty$) — $H = F = 2/3\alpha l$, and hence

$$|Q| = \lambda \frac{|\tilde{T}_1|}{l/\sqrt{2}} \left[1 + \frac{8}{3} \frac{\sigma n^2 \overline{T}_1^3}{\alpha\lambda}\right]. \tag{12}$$

This formula corresponds to the Rosseland approximation, for which it is possible, without the need to solve the problem of radiative–conductive heat transfer, to solve the problem of energy transfer by heat conduction alone and to replace the heat conduction λ by the effective value $\lambda^* = \lambda + \lambda^r$, where $\lambda^r = 16n^2\sigma\overline{T}^3/3\alpha$ is the so-called radiant heat conduction. It is also necessary to assume that Bi = 0 in Eq. (8) and to take into account that $l \sim \sqrt{\lambda}$. Physically, this means that at large αl (large effective Knudsen number), radiant heat transfer becomes a local process, and the vector $\vec{q}$ is linearly related to the temperature gradient (the gradient approximation).

Taking into account Eqs. (6) and (7), Eq. (1) for the foil heat balance takes the form

$$\frac{W_0}{s} = \frac{cm}{s} \omega\tilde{T}_1 i + 2b\sqrt{\omega}\,\tilde{T}_1 i(1 + \mathrm{Bi}\,H) + 2b\sqrt{\omega}\,\tilde{T}_1(1 + \mathrm{Bi}\,F). \tag{13}$$

Algebraic manipulations in Eq. (13) yield the amplitude of the foil temperature pulsations, which is given by the expression

$$|\tilde{T}_1| = \frac{W_0/2s\sqrt{\omega}}{\sqrt{d^2 + 2db(1+\mathrm{Bi}\,H) + b^2[(1+\mathrm{Bi}\,H)^2 + (1+\mathrm{Bi}\,F)^2]}}. \tag{14}$$

Curves of F and H are shown in Fig. 2. Setting Bi = 0 or H = F = 0 reduces Eq. (14) to the formula obtained when no account is taken of radiant energy transfer [1].

In experimental determinations of the thermal activity of a medium by electrical methods, the amplitude of the temperature pulsations $|\tilde{T}_1|$ is measured. The thermal activity can be calculated from Eq. (14) without taking radiation into account if the sensor (foil) properties, the power input W_0, and the electric-current frequency ω are known.

An estimate of the contribution to the measured value of b by radiant energy transfer may be obtained by finding the difference (denoted by Δb) between the thermal activities calculated from Eq. (14) with and without taking account of radiation. Assuming that $\Delta b \ll b$, it is simple to find that

$$\frac{\Delta b}{b} = \mathrm{Bi}\,\frac{b(H+F) + Hd}{2b+d}. \tag{15}$$

To obtain a numerical estimate of the role of radiation, consider a typical case, a material with the following thermophysical and optical properties (toluene at 30°C): $\lambda = 0.13$ W/m·deg, $c_p = 1700$ J/kg·deg, $\rho = 860$ kg/m^3, n = 1.5. The sensor is assumed to be inertialess (d = 0).

Results of the calculations for T = 300°K are shown in Fig. 3. At higher temperatures, if the thermophysical and optical properties are largely unchanged, the values of $\Delta b/b$ in Fig. 3 must be multiplied by $(\overline{T}_1/300)^3$.

The most notable feature of Fig. 3 is the sharp decrease in $\Delta b/b$ with increase in frequency, especially in the low-frequency region. This is valid for media that do not absorb radiation strongly ($\alpha \leqslant 10^5$). For strongly absorbing media ($\alpha \geq 3 \cdot 10^5$), the proportion of radiation in the low-frequency region is considerably less than for transparent and semitransparent media, and the frequency dependence is expressed much more weakly. Estimates of the specific values of $\Delta b/b$ lead to the conclusion that, in the given method of investigating weakly absorbing and transparent media, the nonsteady method of measuring the thermal properties of the material is considerably preferable to the steady-state method, since the correction to the radiation for the nonsteady method in the experimentally convenient frequency range (20-1000 Hz) is one to two orders of magnitude less at the same temperatures. Physically, this means that, in periodic heating even at low frequencies, temperature differences are produced close to the sensor surface at small distances, which are difficult to realize in the steady method (this requires too narrow a gap: for toluene at 30°C and 20 Hz, $l = 27\ \mu$). Therefore, the heat flux is much larger with respect to the radiant flux in the nonsteady method than in the steady-state method, and increases with frequency.

As already noted, in steady methods to determine the fraction of the energy transferred by radiation, it is necessary to vary the gap between the plates producing the heat flux through the sample material, which involves very great complication both of the apparatus and of the experiment itself in the case of experiments at high temperatures. In the nonsteady method, the fraction of radiant energy may be determined, in principle, by varying the frequency. This is only possible in practice in the low-IR region, where the fraction of radiant energy is of a measurable size. Special measuring equipment must be developed for such an experiment. In the available apparatus at frequencies above 20 Hz, radiation may only be detected at high temperatures.

NOTATION

λ, heat conduction of the medium; ρ, density of the medium; c_p, specific heat at constant pressure; $b = \sqrt{\lambda c_p \rho}$, coefficient of thermal activity of medium; c, specific heat of foil material; m, foil—sensor mass; s, foil surface area (one side); W, power input to foil by electrical heating; ω, angular frequency of a current heating foil; x, y, z, rectangular coordinates; T(x), temperature of medium; $\bar{T}$, mean temperature of medium; $\tilde{T}(x)$, complex amplitude of temperature pulsations of medium; T_1, foil temperature; $\bar{T}_1$, mean foil temperature; $\tilde{T}_1$, complex amplitude of temperature pulsations of foil; q(x), radiant energy flux in medium; Q, radiative—conductive energy flux from foil surface; l, damping length of temperature wave; σ, Stefan—Boltzmann constant; n, refractive index of medium; α, absorption coefficient of medium; $Bi = 4\sigma n^2 \bar{T}_1^3 l/\lambda$, Biot number; i, imaginary unity; R, reflective index of foil surface; $d = cm\sqrt{\omega}/s$, parameter characterizing heat-inertia properties of foil.

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EXPERIMENTAL STUDY OF SPECIAL FEATURES OF HEAT AND MASS TRANSFER IN A TWO-COMPONENT LOW-TEMPERATURE HEAT PIPE

Ya. M. Baum, V. P. Sorokin,* and S. S. Yurov

UDC 536.27.001.5

This paper presents results of an experimental investigation of the characteristics of a two-component heat pipe, operating with a mixture of water and ethanol.

A number of reports [1-7] have been published dealing with theoretical and experimental investigations of heat pipes in which a liquid mixture is used as the heat-transfer agent. Interest in two-component heat pipes arises mainly from the following causes:

* Deceased.

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